Homework Problems
Physics 417/517
Due December 10, 2009

1. What is the total radiation power from the bends by a 100 mA electron beam current stored in the Advanced Photon Source (APS)? Recall from last homework that the beam energy is 7 GeV and the bending radius is 38.96 m. Hint: the total beam power is 100 mA × 7 GV = 700 MW.

2. How much power is emitted, in the lab frame, from an APS "Undulator-A" operated with $K = 1$ by an electron beam current of 100 mA at 7 GeV. Recall the undulator period is 2.3 cm, and the undulator has 72 periods. If there are 50 such devices around the storage ring, how does this power compare with the result of problem 1?

3. What is the fundamental damping time, in msec, from the bend radiation from the APS. To make this estimate, you will need the revolution time, which may be found at the APS site:
   http://www.aps.anl.gov/Facility/Storage_Ring_Parameters/node1.html
   Note from the document that the partition constant is small. Calculate and verify the individual damping rates for the betatron oscillations and synchrotron oscillations.

4. Derive the expression for the damped energy spread in a storage ring
   $$\frac{\sigma_E^2}{E^2} = \frac{55}{32\sqrt{3}} \frac{\hbar c}{mc^2} \frac{\gamma^2}{2+\vartheta} \frac{\langle 1/\rho^3 \rangle}{\langle 1/\rho^2 \rangle}$$
   where the bend contribution alone is considered. The following Bessel Function integral is needed $\int_0^\infty \xi^3 K_{5/3}(\xi)\, d\xi = \frac{55\pi}{27}$.

5. Verify the correct number of photons is obtained by integrating the lab-frame photon energy spectrum $\int_{1/(1+\beta)}^{1/(1-\beta)} \frac{dN_\gamma}{d\hat{E}} d\hat{E} = \frac{2\pi}{3} \alpha N K^2$. Furthermore, verify that the correct value for the average energy is obtained by integrating (the integrals are straightforward but take a little time to complete!)
   $$\langle E_\gamma \rangle = \int_{1/(1+\beta)}^{1/(1-\beta)} E \frac{dN_\gamma}{d\hat{E}} d\hat{E} \Big/ \int_{1/(1+\beta)}^{1/(1-\beta)} \frac{dN_\gamma}{d\hat{E}} d\hat{E}.$$
   Finally, calculate the *rms* spread of the emitted photons, which is $\sqrt{\langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2}$.
   Hint: before starting this problem evaluate the integrals
   $$\int_{1/(1+\beta)}^{1/(1-\beta)} \hat{E}^n d\hat{E}$$
   for n from 0 to 4.